RAYLEIGH WAVE CORRECTION FOR THE BEM ANALYSIS OF TWO-DIMENSIONAL ELASTODYNAMIC PROBLEMS IN A HALF-SPACE

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The BEM formulation for problems of wave propagation in a half-space is usually stated in terms of full-space Green's functions. Thus, the discretization over the boundary of the half-space is needed, and its infinite extent requires a special treatment. We present a simple, elegant approach to the treatment of infinite boundaries for time-harmonic problems. The formulation is detailed for a two-dimensional, homogeneous, isotropic, linearly elastic half-space in which the main objective is to allow the undamped Rayleigh waves to propagate to infinity. However, the basic ideas are applicable to a broader range of wave propagation problems, such as those involving Stoneley waves in material interfaces, or propagating Lamb modes in layers.

The proposed method consists of two parts. First, common to other techniques [1], it is assumed that the numerical solution takes the known far-field general form of Rayleigh waves in the omitted part of the boundary of unknown amplitude and phase. Thus, the integrals over the omitted part of the boundary are rewritten as the product of the complex amplitude and integrals of the known far-field Rayleigh wave. The assumed far-field Rayleigh waves are matched to the nodal values at the end nodes of the computational boundary, thereby modifying the original BEM displacement matrix. Next, the integrals of the known far-field Rayleigh wave on the infinite omitted part of the boundary are computed. These integrals may be approximated numerically [2]. By contrast, the reciprocity theorem of elastodynamics is invoked here to derive a boundary integral representation for the known far-field Rayleigh wave with the same fundamental solution of the original formulation [3]. This representation involves the same integrals over the omitted part of the boundary that are needed to modify the original BEM system, integrals of known quantities on the originally discretized part of the boundary and in some cases integrals on additional boundaries whose computational cost is very small. This simple approach allows for a very efficient numerical implementation in terms of the same basic element integrals of the original BEM scheme, and it comes at a very low, in many cases negligible, additional cost as compared to the simple truncation of the boundary.

The proposed method is validated for a free Rayleigh wave propagating in a half-space and tested for a benchmark problem with a known asymptotic approximation to the analytical solution. For the present method to be accurate, the discretized boundary needs to be extended far enough for the body waves to have substantially attenuated. In return, no spurious reflections are produced, and the accuracy of the solution is not degraded near the ends of the mesh.

References

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